Attribute-Based Broadcast Encryption Scheme Made Efficient

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Outline

1. Context
   - Broadcast encryption
   - Efficiency of standard schemes
   - Attributes in broadcast encryption
   - A previous construction

2. Our Scheme
   - Principle of the scheme
   - In practice ?
   - Full description of the scheme
   - Efficiency and security

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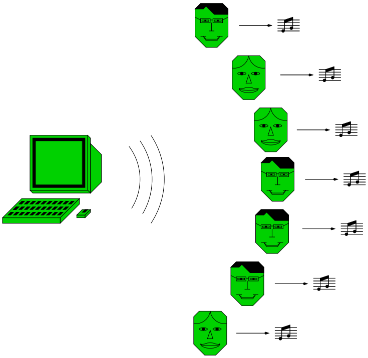
   - Full description of the scheme
   - Efficiency and security

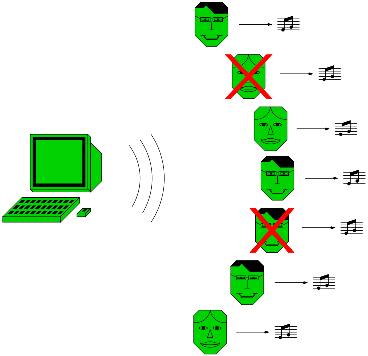
Broadcast

A broadcaster intends to send securely and efficiently the same message to a large number of receivers:



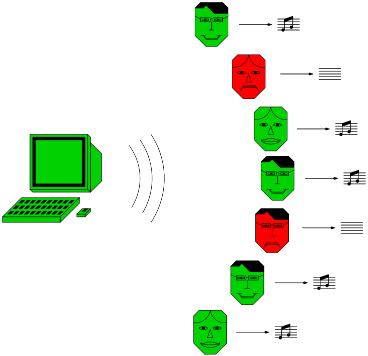
Revocation

In the case of a revocation, some users are removed from the set of receivers (for example when their decryption keys are compromised):



Permanent revocation

In the case of a permanent revocation, the decryption keys are updated. The revoked users are not able anymore to obtain the messages sent by the broadcaster:



Temporary revocation

When permanent revocations are used (stateful schemes),

- receivers must remain online,
- receivers must store and use new decryption keys.

To avoid these limitations, it is possible to use stateless schemes, where revocations are temporary:

- in the encryption process, the broadcaster chooses the set of receivers,
- only members of this set may decrypt the message.

Efficiency of standard schemes

In stateful schemes (like LKH),

- a common decryption key is known by all receivers,
- a specific structure allows join and revoke operations.

$\rightarrow$ Join and revoke operations require large bandwidth.

▶ Good schemes for sets of receivers with rare modifications.

In stateless schemes (like CS or SD),

- join and revoke operations do not exist,
- a specific structure allows encryption.

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Attributes and users

In practical applications of broadcast, users have attributes that can be used to describe efficently the set of receivers.

Id	Name	Subscription	Expiry Date	Location
1	Alice	Movies	Jan 2009	Europe
2	Bob	News / Sports	Aug 2009	Africa
3	Charlie	Entertainment	May 2008	Asia
4	Dave	News / Movies	Jun 2009	Asia
5	Eve	Entertainment / Sports	Jan 2008	Africa

Is it possible to send efficiently a movie in June 2008 ?

- Evolution of the set of receivers : fast
- Number of revoked users: large
- Number of users: large
- But a specific structure !

Goal

Build a broadcast encryption scheme such that:

- when the set of receivers is defined by attributes, the efficiency depends only on the number of attributes used,
- any set of receivers has a "reasonnable" efficiency ?

A previous construction

An attribute-based broadcast scheme comes from:

- [SW05]: use of a single attribute,
- [GPSW06]: use of several attributes,
- [BSW07]: policy defined by the ciphertext,
- [OSW07]: non-monotonic policy (use of NOT).

The policy is defined by:

- threshold functions (including AND and OR functions),
- NOT functions.

Efficiency:

++ Ciphertexts have a linear size in the number of attributes,

−− Decryption requires a linear number of pairing computations.

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Outline

Principle of the scheme (1)

Each attribute is associated with an element $\mu_i \in \mathbb{Z}/p\mathbb{Z}$.

User u:

- $\Omega(u)$ is its set of attributes,
- dk_u is its decryption key.

$$\mathrm{dk}_u \quad \longleftrightarrow \quad \prod_{\mu \in \Omega(u)} (X - \mu).$$

Encryption: a header hdr and a key K are built from

- Ω^R: the set of revoked attributes,
- Ω^N: the set of needed attributes.

$$\mathrm{K} \longleftrightarrow \prod_{\mu \in \Omega^N} (X - \mu) \qquad \mathrm{hdr} \longleftrightarrow \prod_{\mu \in \Omega^R \cup \Omega^N} (X - \mu).$$

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Principle of the scheme (2)

Decryption: compute the GCD (greatest common divisor) of the decryption key dk_u and the header hdr to obtain the key K.

$$\mathsf{GCD}(\mathrm{dk}_u, \mathrm{hdr}) \quad \longleftrightarrow \quad \prod_{\mu \in \Omega(u) \cap (\Omega^R \cup \Omega^N)} (X - \mu).$$

Is it accurate ?

$$\begin{aligned} & \mathsf{GCD}(\mathrm{dk}_u, \mathrm{hdr}) = \mathrm{K} \\ \Longleftrightarrow \quad & \Omega(u) \cap (\Omega^R \cup \Omega^N) = \Omega^N \\ \Longleftrightarrow \quad & \Omega(u) \cap \Omega^R = \emptyset \ \text{ and } \ \Omega^N \subset \Omega(u). \end{aligned}$$

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In Practice ?

In practical applications, it is not possible to use polynomials.
For all polynomial P, we use $P(\alpha)\, g_1$ instead of P, where:

- g_1 is a public generator of a group G_1 in which the DLP is hard,
- α is a secret value.

The "GCD" is computed using extended Euclide's algorithm.
We need a non-degenerate pairing, i.e. a map $e : G_1 \times G_1 \to G_2$ where:

- (G_1, g_1) and (G_2, g_2) are two cyclic groups of same prime order p,
- $e(g_1, g_1) = g_2$,
- e is bilinear.

The group laws in G_1 and G_2 are noted additively: $e(a\, g_1, b\, g_1) = a\, b\, g_2$.

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Protection against some attacks

Attack 1 : Attributes in headers can be modified.
(Linear combinations of different headers)
$\hookrightarrow$ Randomized headers (using z).

Attack 2 : Attributes in decryption keys can be modified.
(Linear combinations of different decryption keys)
$\hookrightarrow$ Randomized decryption keys (using s_u).

Attack 3 : Other computations (than "GCD") can be performed.
(Pairing computations on some specific pairs of group elements)
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Full scheme - key generation

- We randomly choose a secret 4-uple $(\alpha, \beta, \gamma, \delta) \in ((\mathbb{Z}/p\mathbb{Z})^*)^4$,
- Each user u is associated with a secret $s_u \in (\mathbb{Z}/p\mathbb{Z})$,
- Each attribute is associated with a public $\mu_i \in (\mathbb{Z}/p\mathbb{Z}) \setminus \{\alpha\}$.

$$\mathrm{EK} = \Big(g_1\,,\, \beta\,\gamma\,\delta\, g_1\,,\, \big(\mu_i\,,\, \alpha^i\, g_1\,,\, \alpha^i\,\gamma\, g_1\,,\, \alpha^i\,\delta\, g_1\big)_{0 \le i \le l} \Big).$$

$$\mathrm{dk}_u = \Big(\Omega(u), (\beta + s_u)\,\delta\, g_1\,,\, \gamma\, s_u\, \Pi(u)\, g_1\,,\, \big(\alpha^i\,\gamma\,\delta\, s_u\, g_1\big)_{0 \le i < l(u)} \Big),$$

$$\text{where} \quad \begin{cases} \Omega(u) = \{\mu_i \in (\mathbb{Z}/p\mathbb{Z}) \,/\, \mu_i \text{ attribute of } u\}, \\ l(u) = |\Omega(u)| \text{ is the number of attributes of } u, \\ \Pi(u) = \prod_{\mu \in \Omega(u)} (\alpha - \mu). \end{cases}$$

Full scheme - encryption

- Let Ω^N be the set of needed attributes.
- Soit $\Omega^R \neq \emptyset$ be the set of revoked attributes.
- A user u is valid for these sets if: $\Omega^N \subset \Omega(u)$ and $\Omega^R \cap \Omega(u) = \emptyset$.

The encryption for these sets (Ω^N, Ω^R) gives :

$$\mathrm{hdr} = \left(\Omega^N, \Omega^R, z\,\Pi^{NR}\,g_1\,,\ \gamma\,z\,\Pi^N\,g_1\,,\ \left(\alpha^i\,\delta\,z\,g_1\right)_{0 \leq i < l^R}\right),$$

$$K = \beta\,\gamma\,\delta\,z\,\Pi^N\,g_2.$$

$$\text{where} \quad \begin{cases} z \text{ is randomly chosen in } (\mathbb{Z}/p\mathbb{Z})^*, \\ l^R = |\Omega^R|, \\ \Pi^N = \prod_{\mu \in \Omega^N} (\alpha - \mu), \\ \Pi^{NR} = \Pi^N \prod_{\mu \in \Omega^R} (\alpha - \mu). \end{cases}$$

Full scheme - decryption

The decryption is based on a decryption key dk_u and a header hdr:

$$\begin{cases} \mathrm{dk}_u = \left(\Omega(u), \mathrm{dk}_1, \mathrm{dk}_2, \mathrm{dk}_{3,0}, \ldots, \mathrm{dk}_{3,l(u)-1}\right), \\ \mathrm{hdr} = \left(\Omega^N, \Omega^R, \mathrm{hdr}_1, \mathrm{hdr}_2, \mathrm{hdr}_{3,0}, \ldots, \mathrm{hdr}_{3,l^R-1}\right). \end{cases}$$

If u is a valid receiver, extended Euclide's algorithm gives two polynomials $V(X) = \sum_{i=0}^{l(u)-1} v_i X^i$ and $W(X) = \sum_{i=0}^{l^R-1} w_i X^i$ such that:

$$V(X) \prod_{\mu \in (\Omega^N \cup \Omega^R)} (X - \mu) + W(X) \prod_{\mu \in \Omega(u)} (X - \mu) = \prod_{\mu \in \Omega^N} (X - \mu).$$

The key is obtained by:

$$e(\mathrm{dk}_1, \mathrm{hdr}_2) - e\left(\sum_{i=0}^{l(u)-1} v_i\, \mathrm{dk}_{3,i}\, ,\ \mathrm{hdr}_1\right) - e\left(\mathrm{dk}_2\, ,\ \sum_{i=0}^{l^R-1} w_i\, \mathrm{hdr}_{3,i}\right).$$

Full scheme - correctness

$$V(\alpha)\Pi^{NR} + W(\alpha)\Pi(u) = \Pi^{N}.$$

$$V(\alpha) = \sum_{i=0}^{l(u)-1} v_i\,\alpha^i \qquad\qquad W(\alpha) = \sum_{i=0}^{l^R-1} w_i\,\alpha^i$$

$$\begin{aligned} \mathrm{dk}_1 &= (\beta + s_u)\,\delta\,g_1 \\ \mathrm{dk}_2 &= \gamma\,s_u\,\Pi(u)\,g_1 \\ \mathrm{dk}_{3,i} &= \alpha^i\,\gamma\,\delta\,s_u\,g_1 \end{aligned} \qquad\qquad \begin{aligned} \mathrm{hdr}_1 &= z\,\Pi^{NR}\,g_1 \\ \mathrm{hdr}_2 &= \gamma\,z\,\Pi^{N}\,g_1 \\ \mathrm{hdr}_{3,i} &= \alpha^i\,\delta\,z\,g_1 \end{aligned}$$

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$$\hookrightarrow \qquad K = \beta\,\gamma\,\delta\,z\,\Pi^{N}\,g_2.$$

Efficiency of this scheme

Size of ciphertexts : linear in $|\Omega^N| + |\Omega^R|$.

Computations :

- Decryption : 3 pairing computations,
- Encryption : 1 pairing computation (none if we extend EK).

Size of keys :

- EK linear in l,
- dk_u linear in $l(u)$.

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Security and other features

The security has been proved in the generic model of groups with pairings.

Some features:

- EK can be strongly reduced in some cases,
- new users can join easily,
- for any set of receivers, at least as efficient as the SD scheme.

Threshold functions are however not available in this scheme.

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▶ Thank you for your attention! ◀