

Differential Equivalences on SBoxes

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Outline

Introduction

Symmetric cryptography

Differentiality

Equivalences

Results

Equivalences

Main contributions

First Step for a classification

Algorithm

Experimental Results

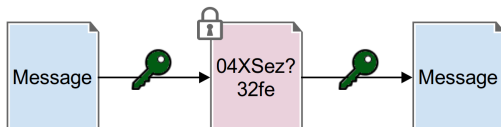
Special case

Conclusion

Symmetric Cryptography

Symmetric Cryptography

Cryptographic **algorithms** and **protocols** where the secret is **identical** between communicating parties.



- ▶ **Block ciphers**
- ▶ Stream ciphers
- ▶ Hash functions

Design of Block ciphers

Block cipher

A **block cipher** is a function

$$\begin{aligned} E : \mathbb{F}_2^k \times \mathbb{F}_2^m &\rightarrow \mathbb{F}_2^m \\ (k, P) &\mapsto C = E(k, P) := E_k(P) \end{aligned}$$

such that for any **fixed key** $k \in \mathbb{F}_2^k$, E_k is *bijective* over \mathbb{F}_2^m .

Problem?

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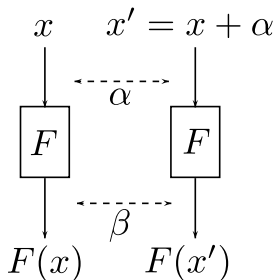
$$\text{SB} : \mathbb{F}_2^m = \mathbb{F}_2^n \times \mathbb{F}_2^n \times \dots \rightarrow \mathbb{F}_2^m = \mathbb{F}_2^n \times \mathbb{F}_2^n \times \dots$$

$$X = (x_0, x_1, \dots) \mapsto (F_0(x_0), F_1(x_1), \dots)$$

Functions F_i 's are usually called *SBoxes*.

Differential Criteria

$$F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$$



Derivatives in direction $\alpha \in \mathbb{F}_2^n$

$$\begin{aligned} \Delta_\alpha F : \mathbb{F}_2^n &\rightarrow \mathbb{F}_2^n \\ x &\mapsto F(x) + F(x + \alpha) \end{aligned}$$

Differential Uniformity [Nyberg 94]

$$\delta_F = \max_{\alpha \in \mathbb{F}_2^n \setminus \{0\}, \beta \in \mathbb{F}_2^n} \# \{x \mid \Delta_\alpha F(x) = \beta\}$$

Whenever $\delta_F = 2$ (**minimal** value), the function F is said to be **APN** ('Almost Perfect Nonlinear').

Example

Differences Distribution Table (DDT)

Table of size $2^n \times 2^n$, with entries:

$$\mathcal{D}_F(\alpha, \beta) = \#\{x \in \mathbb{F}_2^n \mid \Delta_\alpha F(x) = \beta\}, \forall \alpha, \beta \in \mathbb{F}_2^n.$$

α	\mathcal{D}_F	β								α	$\mathcal{D}_{F'}$	β							
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
	0	8		0	8
	1	.	2	.	2	.	2	.	2		1	.	.	8
	2	.	.	2	2	.	.	2	2		2	.	.	.	4	.	.	4	.
	3	.	2	2	.	.	2	2	.		3	.	2	.	.	6	.	.	.
	4	2	2	2	2		4	2	.	2	4
	5	.	2	.	2	2	.	2	.		5	.	2	.	2	.	.	2	2
	6	.	.	2	2	2	2	.	.		6	.	2	.	2	.	4	.	.
	7	.	2	2	.	2	.	.	2		7	.	2	.	.	.	4	.	2

Big APN Problem

APN functions \Rightarrow Best resistance to **differential cryptanalysis!**

1. **Very few** known APN functions . . .
2. . . . even fewer known **bijjective** APN functions (none when $n = 4$)

'Big APN Problem' :

Is there any **bijjective APN functions** over \mathbb{F}_{2^n} , when n is **even** ?

Answer (?) : In 2009, Dillon et al. found **only one** example for $n = 6$. Since then, **nothing more** (or very little) . . .

3. **Few** (bijjective) functions are known with $\delta = 4$.



K. A. Browning, J. F. Dillon, M. T. McQuistan and A. J. Wolfe,
An APN Permutation in Dimension Six,
Fq9 (Selected Papers), Contemporary Mathematics, 2010.

Some equivalence classes

$$F : \mathbb{F}_{2^n} \rightarrow \mathbb{F}_{2^n}$$

The **differential uniformity** $\delta(F)$ of F is an **invariant** under:

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 \Rightarrow **EA-equivalence** (\sim_{EA})

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- ▶ Graph equivalence: $\mathcal{G}_{F'} = \{\mathcal{L}(x, F(x))\} = \mathcal{L}(\mathcal{G}_F)$, $\delta(F) = \delta(F')$
 \Rightarrow **CCZ-equivalence** (\sim_{CCZ})

$$\mathcal{M}\mathcal{G}_F = \begin{pmatrix} \begin{array}{c} | \\ 1 \\ | \end{array} & \dots & \begin{array}{c} | \\ x \\ | \end{array} & \dots & \begin{array}{c} | \\ 2^n - 1 \\ | \end{array} \\ \hline \begin{array}{c} | \\ F(1) \\ | \end{array} & \dots & \begin{array}{c} | \\ F(x) \\ | \end{array} & \dots & \begin{array}{c} | \\ F(2^n - 1) \\ | \end{array} \end{pmatrix}$$

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$$\text{EA-Eq} \xRightarrow{\quad} \text{CCZ-Eq}$$

Link with other areas

$$F : \mathbb{F}_{2^n} \rightarrow \mathbb{F}_{2^n}$$

Carlet-Charpin-Zinoviev (1998)

F is **APN if and only if** the **minimal distance** of the linear code with **parity check matrix** \mathcal{MG}_F is 5.

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If F is **APN** then the **incidence structure** of \mathcal{G}_F is a $(2^{2n}, 2^n)$ -**semiplane**.

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Let F be **quadratic**, i.e. $F(x) = \sum_{i,j} c_{i,j} x^{2^i+2^j} \in \mathbb{F}_{2^n}[x]$.

F **APN** \Rightarrow **semifields, dual hyperoval, rank-metric code,...**

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⋮

not only **cryptography** benefits from research on
APN functions.

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Definitions :

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Table of size $2^n \times 2^n$, with entries:

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Indicator of a DDT

Boolean function $\gamma_F : \mathbb{F}_2^{n \times n} \rightarrow \mathbb{F}_2$,

$$\gamma_F(\alpha, \beta) = 0 \Leftrightarrow \delta_F(\alpha, \beta) = 0 \text{ or } \alpha = 0.$$

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\Rightarrow Two equivalences:

$$\blacktriangleright \quad F \sim_{\mathcal{D}} G \quad \Leftrightarrow \quad \mathcal{D}_F = \mathcal{D}_G,$$

$$\blacktriangleright \quad F \sim_{\gamma} G \quad \Leftrightarrow \quad \gamma_F = \gamma_G.$$

Motivation, Origin

Differential Equivalence

In **odd** characteristic, **Perfect Nonlinear** (PN) functions are such that all of their derivatives are **bijective** \Rightarrow all PN functions have the **same DDT**.

Gorodilova introduced the **γ -equivalence**, while working on **APN monomial quadratic** functions (aka **Gold** functions, aka APN functions of the type $x \mapsto x^{2^i+1}$).



A Gorodilova,

On a remarkable property of APN Gold functions,
Cryptography ePrint Archive, 2016.

Examples

$n=4$

$$F = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 14],$$

$$G = [0, 1, 3, 2, 5, 4, 7, 6, 8, 9, 10, 11, 12, 13, 14, 15].$$

$$D_F = \begin{bmatrix} 16 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & 16 & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & 12 & 4 & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & 4 & 12 & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & 12 & 4 & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & 4 & 12 & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & 12 & 4 & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & 4 & 12 & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & 12 & 4 & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & 4 & 12 & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & 12 & 4 & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & 4 & 12 & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & 12 & 4 & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & 4 & 12 & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & 12 & 4 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & 4 & 12 \end{bmatrix}$$

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$$\gamma_G = \begin{pmatrix} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 \end{pmatrix} = \gamma_F$$

Properties

$$F, G : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$$

Basic:

$$\blacktriangleright F \sim_{\mathcal{D}} G \Rightarrow F \sim_{\gamma} G \quad (\not\Leftarrow)$$

\blacktriangleright for any $a, b \in \mathbb{F}_2^n$,

$$F(x) \sim_{\mathcal{D}} G(x) = F(x + a) + b$$

in that case, F and G are said to be **trivially equivalent**.

A little more advanced:

\blacktriangleright if F and G are either **APN** or **quadratic** (or both), then

$$F \sim_{\mathcal{D}} G \Leftrightarrow F \sim_{\gamma} G$$

What are the functions that share the same **DDT**?
the same **indicator**? How many of them exist?

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Contributions :

- Using **EA-Eq** and **CCZ-Eq** to *help* the **classification**
- a **recursive algorithm** to compute **equivalence classes**.
Input : a **DDT** (resp. **indicator**),
Output : **every functions** having this **DDT** (resp. **indicator**)
- **research** for some **known functions**.
- **new fact** about **APN permutations**.



C. Boura, A. Canteaut, J. Jean and V. Suder,
Two Notions of Differential Equivalence on SBoxes,
DCC, Design, Codes and Cryptography, To appear.

Relation to CCZ-Eq and EA-Eq

Gorodilova (2016)

For $F \sim_{EA} G$,

$$\forall F' \sim_{\gamma} F \Rightarrow \exists G' \sim_{\gamma} G \text{ s.t. } G' \sim_{EA} F'$$

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For $F \sim_{CCZ} G$,

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Input : a DDT (resp. indicator),

Output : **every functions** having this DDT (resp. indicator)

Idea: Recursive Tree-traversal algorithm

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- ▶ Tree of depth 2^n , each nodes at a level i corresponds to one **possible value** for $F(i)$
- ▶ From the constraints of the DDT and the values $F(0), \dots, F(i-1)$:
 - find **all possible values** for $F(i)$
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Pruning trick: We can **fix** $F(0)$ and $F(1)$:

$$G(x+1) + G(1) := F(x) \sim_{\mathcal{D}} G(x)$$

so that

$$F(0) = 0, \text{ and } F(1) = \Delta_1 F(0) = \Delta_1 G(0)$$

Example

 \mathbb{F}_2^3

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
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1. $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$
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2. $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$

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Set $F(1) = 1$
2. $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$

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3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set $F(0) = 0$
1. $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$
Set $F(1) = 1$
2. $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ thus
 $F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

Example

 \mathbb{F}_2^3

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

$$3. F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$$

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0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$

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\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
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0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

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 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
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Example

 \mathbb{F}_2^3

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\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

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1. $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

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2. $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$ thus

$F(3) \in \{2, 6\}$ (OR $\{2, 6\}$)

4. $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$

Example

 \mathbb{F}_2^3

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set $F(0) = 0$

1. $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set $F(1) = 1$

2. $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$ thus

$F(3) \in \{2, 6\}$ (OR $\{2, 6\}$)

4. $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$ and
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$

Example

 \mathbb{F}_2^3

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set $F(0) = 0$

1. $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set $F(1) = 1$

2. $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$ thus

$F(3) \in \{2, 6\}$ (OR $\{2, 6\}$)

4. $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$ and
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$ and
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$

Example

 \mathbb{F}_2^3

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

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1. $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

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2. $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$ thus

$F(3) \in \{2, 6\}$ (OR $\{2, 6\}$)

4. $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$ and
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$ and
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$ and
 $F(3) + F(4) \in \mathcal{R}_7 = \{1, 2, 4, 7\}$

Example

 \mathbb{F}_2^3

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

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1. $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set $F(1) = 1$

2. $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$ thus

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 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$ and
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$ and
 $F(3) + F(4) \in \mathcal{R}_7 = \{1, 2, 4, 7\}$ thus

$F(4) \in \{\emptyset\}$ if $F(2) = 3$ and $F(3) = 2$

Example

 \mathbb{F}_2^3

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

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 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ thus

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3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
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 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$ and
 $F(3) + F(4) \in \mathcal{R}_7 = \{1, 2, 4, 7\}$ thus

$F(4) \in \{\emptyset\}$ if $F(2) = 3$ and $F(3) = 2$

$F(4) \in \{7\}$ if $F(2) = 3$ and $F(3) = 6$

$F(4) \in \{5\}$ if $F(2) = 7$ and $F(3) = 2$

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Example

 \mathbb{F}_2^3

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\mathcal{D}	0	1	2	3	4	5	6	7
0	8
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

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3. $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$ and
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$ and
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$ thus

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 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$ and
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$ and
 $F(3) + F(4) \in \mathcal{R}_7 = \{1, 2, 4, 7\}$ thus

$F(4) \in \{\emptyset\}$ if $F(2) = 3$ and $F(3) = 2$

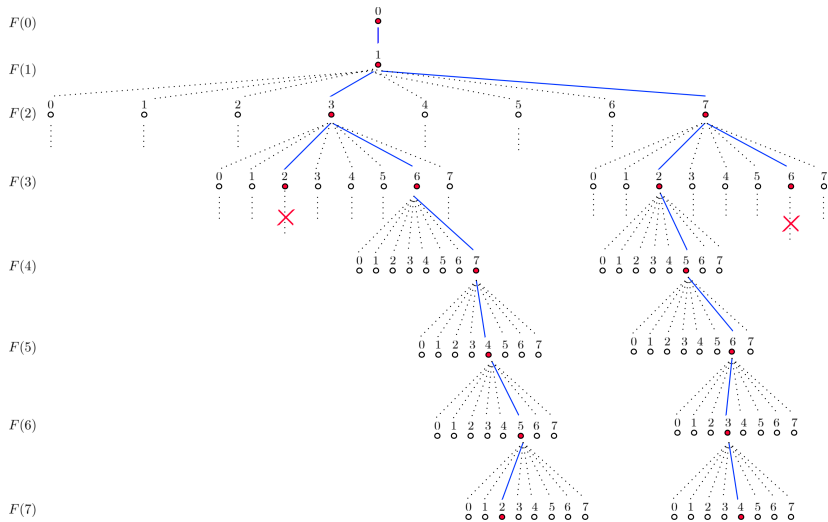
$F(4) \in \{7\}$ if $F(2) = 3$ and $F(3) = 6$

$F(4) \in \{5\}$ if $F(2) = 7$ and $F(3) = 2$

$F(4) \in \{\emptyset\}$ if $F(2) = 7$ and $F(3) = 6$

5. $F(0) + F(5) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$

\vdots



Known functions

F and G are *trivial* if $F(x) \sim_{\mathcal{D}} G(x) = F(x + a) + b$

[Gorodilova'16] \rightarrow (non-bijective) APNs with **non-trivial** γ -classes.

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What are the DDT-classes of '*optimal*' bijective SBoxes?

- ▶ $n = 6$: Dillon permutation \rightarrow **trivial** DDT-class
- ▶ $n = 5$: five APN permutations \rightarrow **trivial** DDT-classes
- ▶ $n = 4$: 16 4-differentially permutations \rightarrow **trivial** DDT-classes
- ▶ $n = 3$: Gold permutation ($x \mapsto x^3$) \rightarrow **trivial** DDT-class

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Question: Are DDT-classes for permutations always trivial? **NO**

Known functions and conjecture

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Question: Are DDT-classes for permutations always trivial? **NO**

Question: Are DDT-classes of non-permutations never trivial? **NO**

Conjecture

For a bijective function,

the **rows** of its DDT are all **distinct** \Rightarrow its **DDT-class** is **trivial**.

Bijjective APN functions and their DDTs

Theorem

Let $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ be a **bijjective APN function**

Then, the rows of its DDT are pairwise **distinct**.

(the image sets of the derivatives of F are pairwise distinct.)

Proof Idea: by contradiction. We suppose two rows match, and then by induction on the images of the derivatives, we show that F is either not APN, or not a permutation.

Bijjective APN functions and their DDTs

Theorem

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Proof Idea: by contradiction. We suppose two rows match, and then by induction on the images of the derivatives, we show that F is either not APN, or not a permutation.

(!!) if the conjecture is **true**, DDT-classes of **APN permutations** are always **trivial**.

Outline

Introduction

- Symmetric cryptography
- Differentiability
- Equivalences

Results

- Equivalences
- Main contributions
- First Step for a classification
- Algorithm
- Experimental Results
- Special case

Conclusion

► **Contributions:**

- Characterize and extend [Gorodilova'16] equivalence
- an algorithm that computes classes
- a new property about APN permutations
- a conjecture on what are the functions with non-trivial classes
- ...

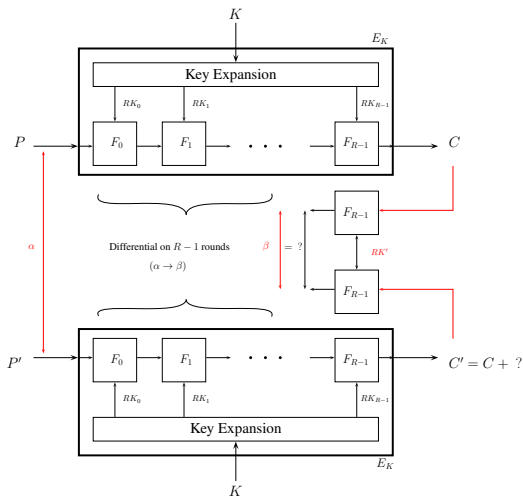
► **Future works:**

- ... prove the conjecture ('easier' cases, e.g. quadratic APN)
- what does it mean for a function to have derivatives with the same image set?
- what are 'possible' DDTs? (use and improve the algorithm)



C. Boura, A. Canteaut, J. Jean and V. Suder,
Two Notions of Differential Equivalence on SBoxes,
DCC, Design, Codes and Cryptography, To appear.

Differential Cryptanalysis of the last round



1. Encrypt N pairs $P, P' = P + \alpha$
2. For each pairs, decrypt the last round with RK'
3. Increment a counter if difference is β
4. Check which counter is the higher (closer from $N \times \mathbb{P}$)



Eli Biham and Adi Shamir,
Differential Cryptanalysis of DES-like Cryptosystems,
J. Cryptology 4(1), 1991.